Lectures on Electromagnetic theory I

PH 2151

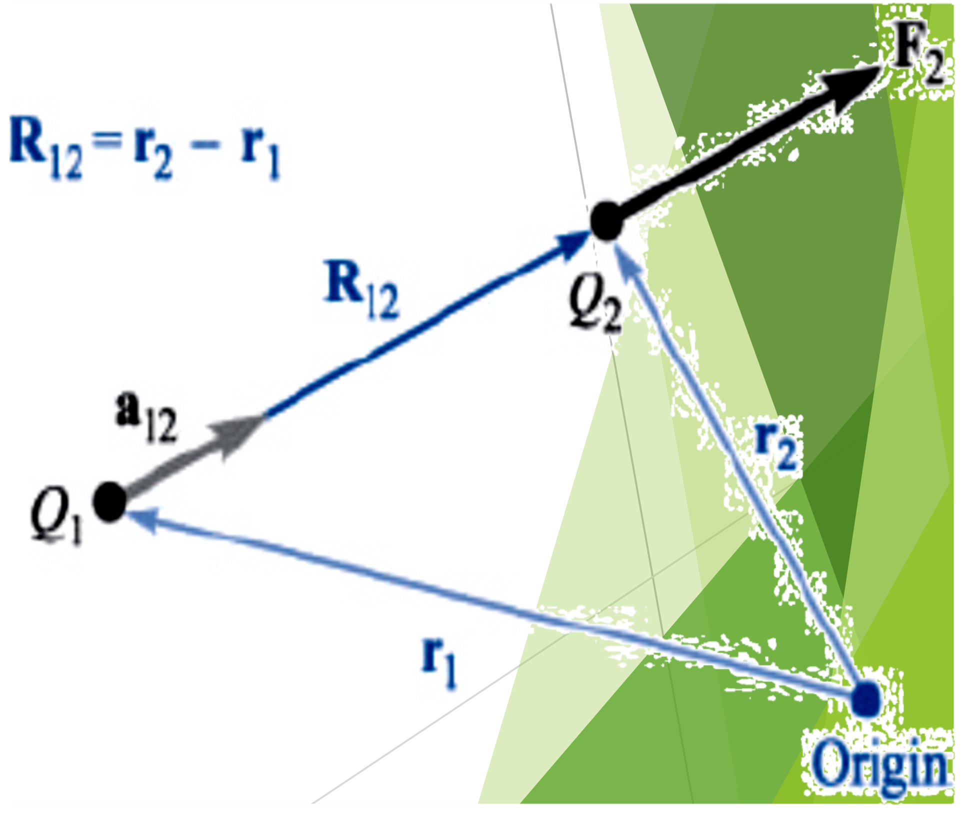
Lecture 3
(Coulomb's law and electric field intensity)

Prof. Salwa Saad Mohamed

Electrostatic field and Coulomb's law

- Producing static charges by rubbing and induction.
- The vector form of Coulomb's law

If Q_1 and Q_2 have like signs, the vector force $\mathbf{F}_2$ on Q_2 is in the same direction as the vector $\mathbf{R}_{12}$



$$\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{12}$$

$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

Point charge electric field intensity E

- E is the vector force on a unite positive test charge
- E units
  newtons per coulomb N/C
  or volts per meter V/m

$$\mathbf{F}_t = \frac{Q_1 Q_t}{4\pi\epsilon_0 R_{1t}^2}\mathbf{a}_{1t}$$

$$\boxed{\mathbf{E} = \frac{\mathbf{F}_t}{Q_t}}$$

$$\mathbf{E} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2}\mathbf{a}_{1t}$$

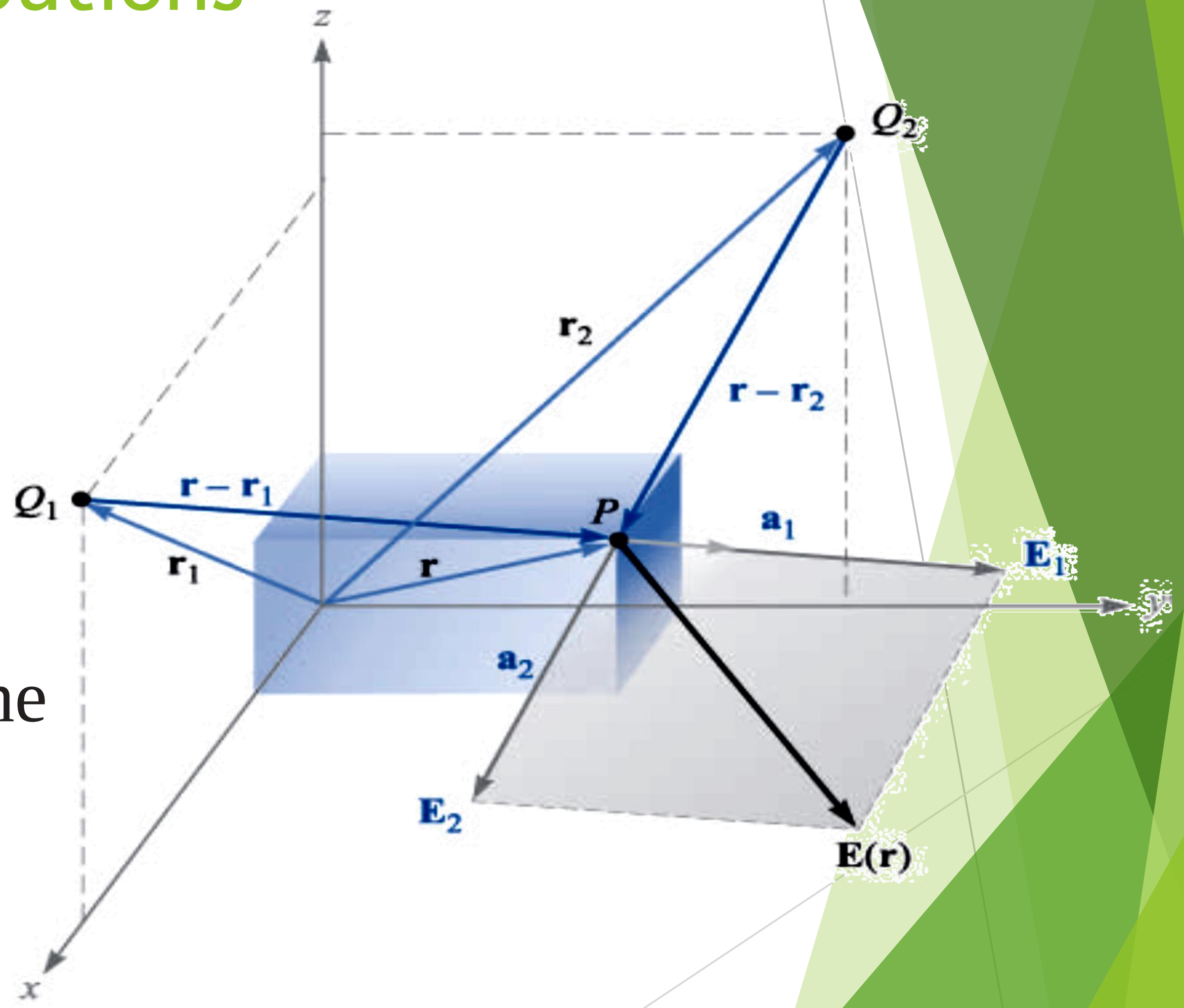
Electric field intensity due to discrete charge distributions

The vector addition of the total electric field intensity at P due to Q_1 and Q_2 is made possible by the linearity of Coulomb's law

$$\mathbf{E}(\mathbf{r}) = \frac{Q_1}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_1|^2}\mathbf{a}_1 + \frac{Q_2}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_2|^2}\mathbf{a}_2$$

Where a_1 and a_2 are unit vectors in the direction of $(r-r_1)$ and $(r-r_2)$ respectively

The vectors $r, r_1, r_2, r-r_1, r-r_2$, a_1 and a_2 are shown in the figure



Electric field intensity due to discrete charge distributions

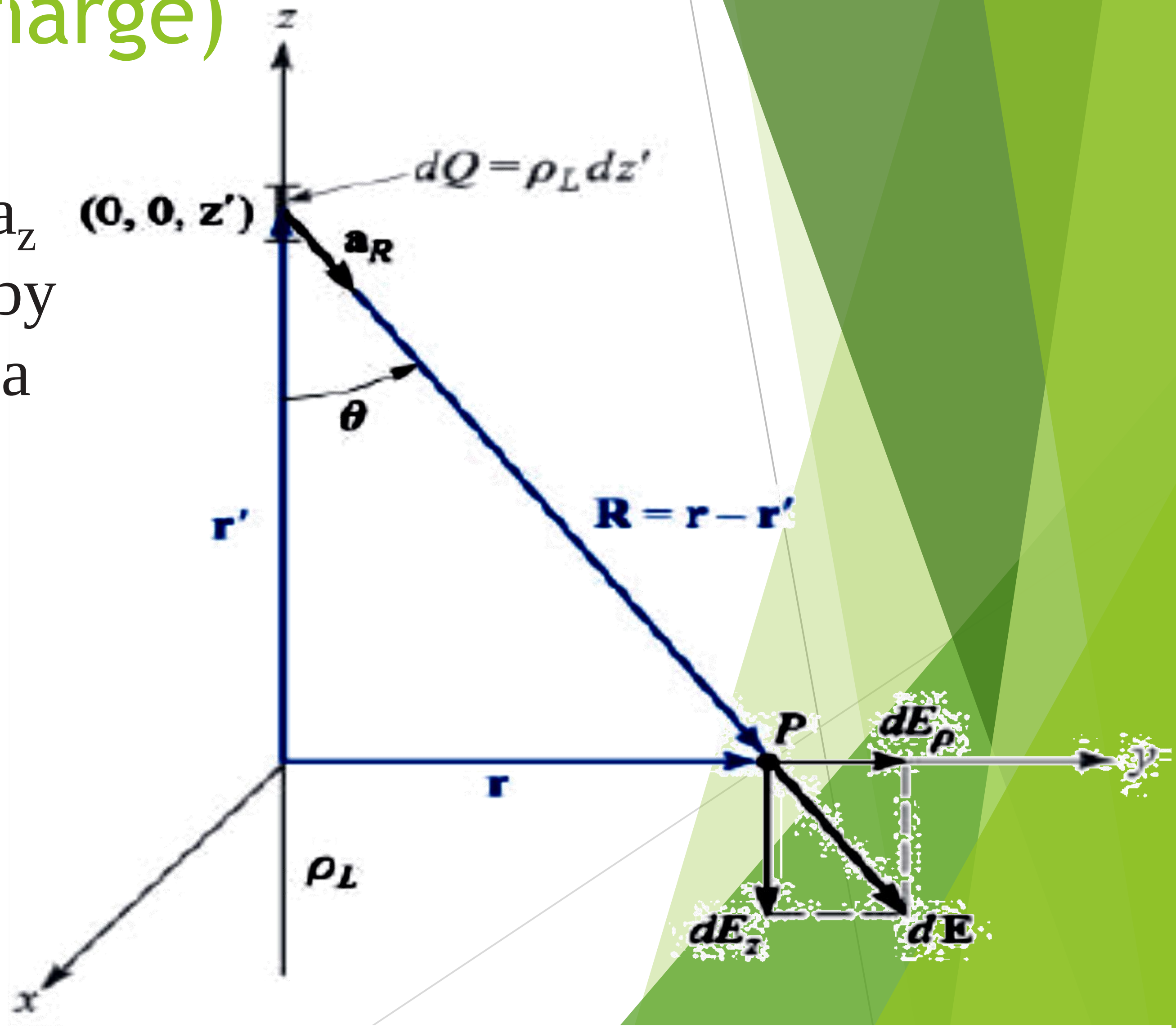
If we add more charges at other positions, the field due to n point charges is

$$\mathbf{E}(\mathbf{r}) = \frac{Q_1}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_1|^2}\mathbf{a}_1 + \frac{Q_2}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_2|^2}\mathbf{a}_2 + \ldots + \frac{Q_n}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_n|^2}\mathbf{a}_n$$

If we add This expression takes up less space when we use a summation sign $\sum$ summing integer m which takes on all integral values between 1 and n

$$\mathrm{E(r)} = \sum_{m=1}^{n} \frac{Q_m}{4\pi\varepsilon_0|r - r_m|^2} a_m$$

Electric field intensity due to a continuous charge distribution (line charge)

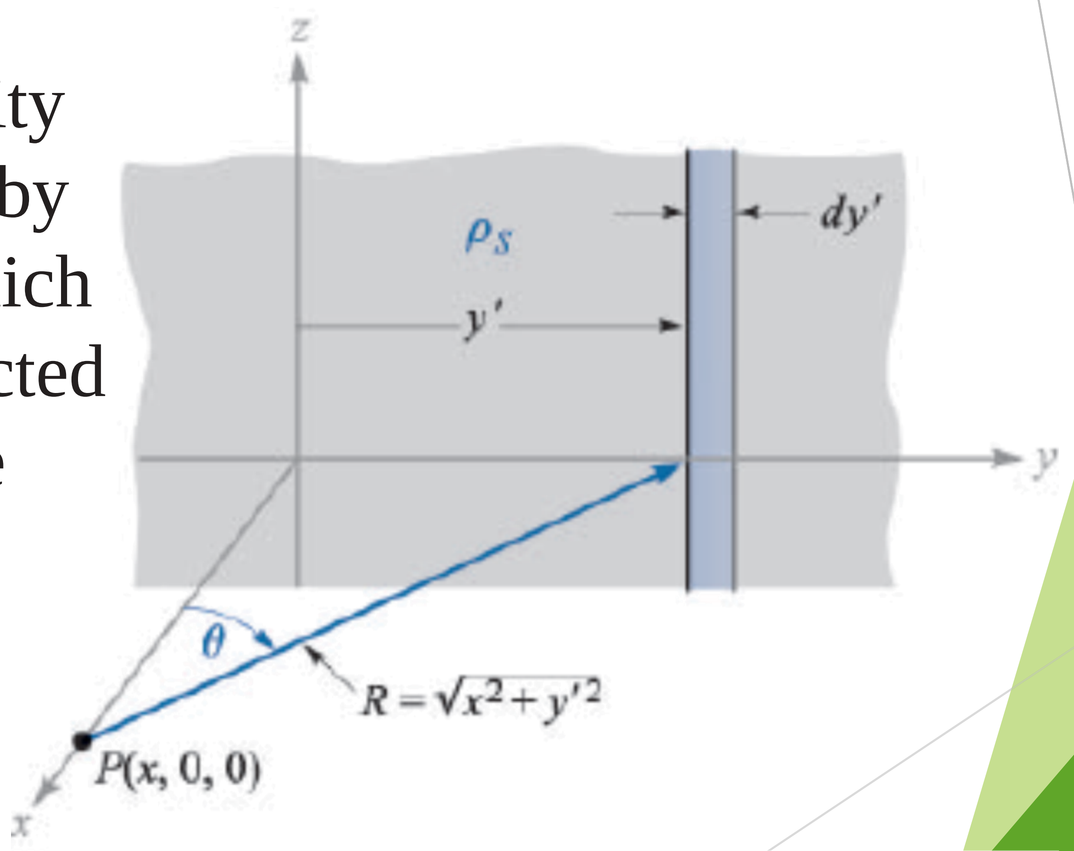


If we The contribution $dE = dE_\rho a_\rho + dE_z a_z$ to the electric field intensity produced by an lement of charge $dQ = \rho_L dz^\backslash$ located a distance $z^\backslash$ from the origin. The linear charge density is uniform and extends along the entire z axis.

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho}\mathbf{a}_\rho$$

Electric field intensity due to a continuous charge distribution (sheet charge)

For an infinite sheet of charge having a uniform surface density ρ_s, the field is always directed by specifying a unit vector a_N, which is normal to the sheet and directed outward and away from it. The electric field intensity is



$$\mathbf{E} = \frac{\rho_S}{2\epsilon_0}\mathbf{a}_N$$

Electric field intensity due to a continuous charge distribution (two sheets of charges “capacitor”)

The field between the parallel plates of an air capacitor, provided the linear dimensions of the plates are very much greater than their separation and provided also that we are considering a point well removed from the edges.

Note: The field outside the capacitor, while not zero, as we found for the ideal case, is usually negligible

$$\mathbf{E}_+ = \frac{\rho_S}{2\epsilon_0}\mathbf{a}_x \qquad \mathbf{E}_- = \frac{\rho_S}{2\epsilon_0}\mathbf{a}_x$$

$$\boxed{\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \frac{\rho_S}{\epsilon_0}\mathbf{a}_x}$$

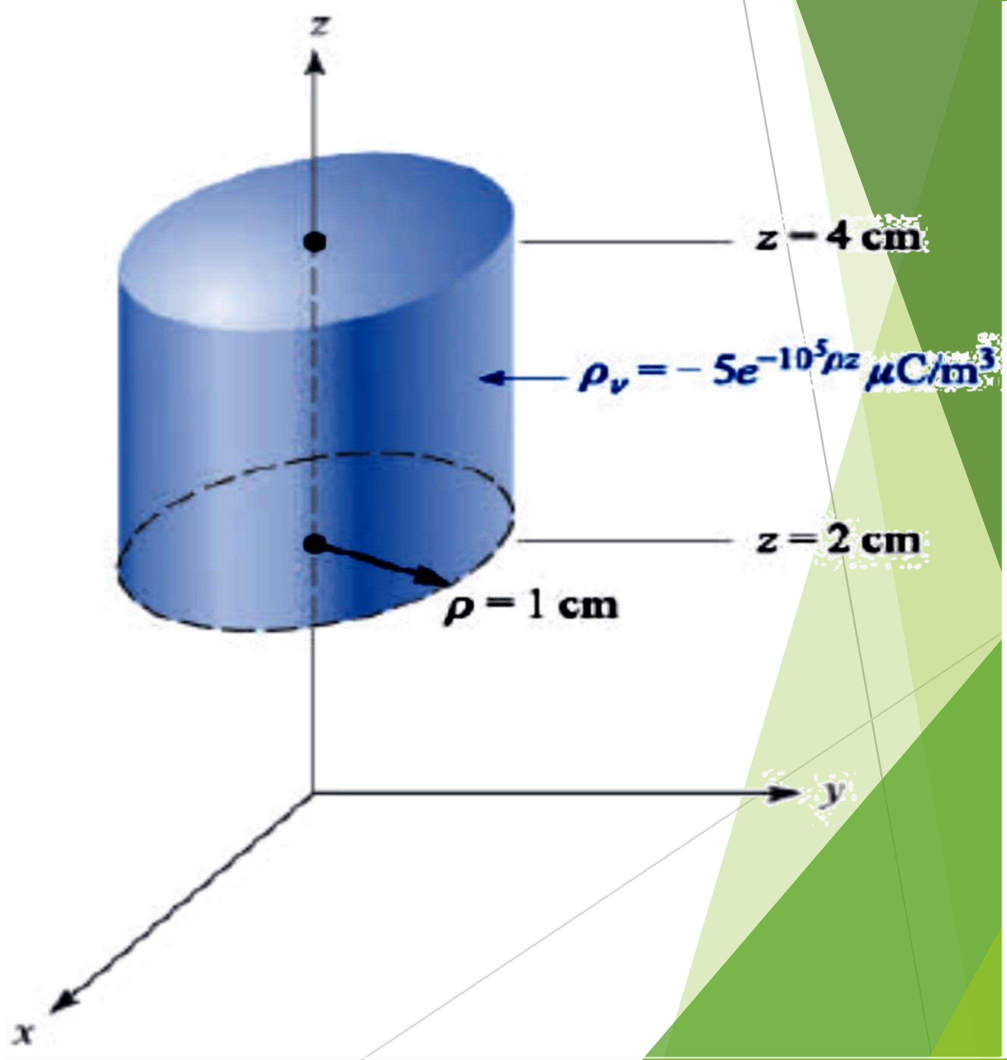
FIELD DUE TO A CONTINUOUS VOLUME CHARGE DISTRIBUTION

Example

The evaluation of the total charge contained in a length of the electron beam.

The total charge contained within the right circular cylinder may be obtained by evaluating

$$Q = \int_{vol} \rho_v \, dv.$$



Problems

1- If the charge Q_1 =20μc at(0,1,2)m. and charge Q_2=-300μc at(2,0,0)m in vacuum. Obtain the force exerted on $Q_1$by Q_2.

2-Obtain E at the point (0,3.4) due to a single positive charge 20 μc at the origin.

3- If the charge Q_1=O.35 μC at (0,4,0)m ,$Q_2 = -O.55$μC at (3,0,0) ,Find E due to the two charges at (0,0,5).

4-IF we have a line charge on z axis extending from -∞ , +∞ with a line charge density ρ_l=20nc/m , find E on the point (6,8,3)m.

5- Circle plate (disk) in the plane z=0, ρ ≤ 1m with charge density $\rho_s = 2(\rho^2 + 25)^{3/2} e^{-10\rho}$. Find E at the point (0,0,5).